

2. SINGLE VS. MULTI POLARIZATION SAR DATA

2.1 Scattering Coefficient vs. Scattering Matrix

In the previous chapter of this document, we dealt with the description and the characterization of electromagnetic waves. As it was shown, one of the main properties of a transverse electromagnetic wave is the vectorial nature of the electromagnetic field, which is called polarimetry.

An electromagnetic wave travels in time and space. In this voyaging through the space, it may happen that the wave can reach a particular target, then interacting with it, see **Figure 1**. As a consequence of this interaction, part of the energy carried by the incident wave is absorbed by the target itself, whereas the rest is reradiated as a new electromagnetic wave. Due to the interaction with the target, the properties of the reradiated wave can be different from those of the incident one. Then, the question which rises at this point is if these changes could be employed to characterize or identify the target. In particular, we are interested in the changes concerning the polarization of the wave. In the following, we present the way in which the interaction of an electromagnetic wave and a given target can be represented.

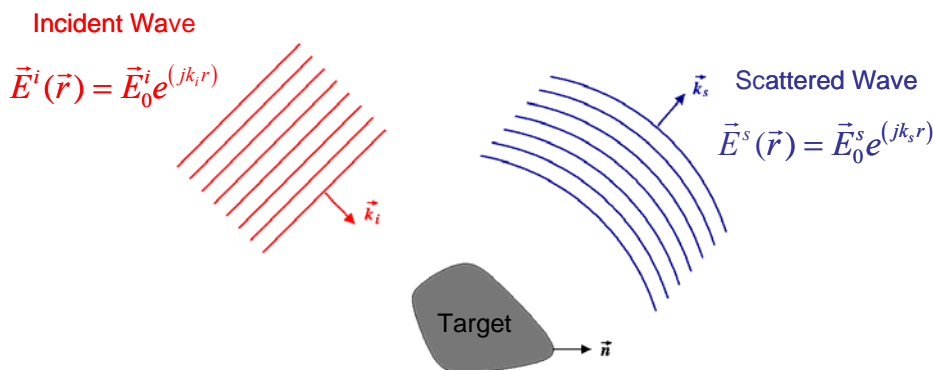


Figure 1 Interaction of an electromagnetic wave and a target.

2.1.1 Single Polarization Image: Scattering Coefficient

Before to define the interaction of electromagnetic waves with the nature, it is necessary to introduce two important concepts concerning the idea of target, since they will determine the way in which they shall be characterized. Given a radar configuration as depicted by **Figure 1**, it may happen the target of interest to be smaller than the coverage of the radar system. In this situation, we consider the target as an isolated scatterer and from a point of view of power exchange, this target is characterized by the so-called *radar cross section*. Nevertheless, we can find situations in which the target of interest is significantly larger that the coverage provided by the radar system. In these occasions, it is more convenient to characterize the

target independently of his extend. Hence, in these situations, the target is described by the so-called *scattering coefficient*.

The most fundamental form to describe the interaction of an electromagnetic wave with a given target is the so-called *radar equation*. This equation establishes the relation between the power which the target intercepts from the incident electromagnetic wave \vec{E}^i and the power reradiated by the same target in the form of the scattered wave \vec{E}^s . The radar equation presents the following form

$$P_r = \frac{P_t G_t}{4\pi R_t^2} \sigma \frac{A_r}{4\pi R_r^2} \quad (1)$$

where P_r represents the power detected at the receiving system. The term

$$\frac{P_t G_t}{4\pi R_t^2} \quad (2)$$

is determined by the incident field \vec{E}^i and it consists of its power density expressed in terms of the properties of the transmitting system. The different terms in (2) are: the transmitted power P_t , the antenna gain G_t and the distance between the system and the target R_t . On the contrary, the term

$$\frac{A_r}{4\pi R_r^2} \quad (3)$$

contains the parameters concerning the receiving system: the effective aperture of the receiving antenna A_r and the distance between the target and the receiving system R_r .

The last term in (1), i.e. σ , determines the effects of the target of interest on the balance of powers established by the radar equation. Since (2) is a power density, i.e., power par unit area and (3) is dimensionless, the parameter σ has units of area. Consequently, σ consists of an effective area which characterizes the target. This parameter determines which amount of power is intercepted from the density (2) by the target and reradiated. This reradiated power is finally intercepted by the receiving system (3), according to the distance R_r . An important fact which arises at this point is the way the target reradiates the intercepted power in a given direction of the space. In order to be independent of this property, the radar cross section shall be referenced to an idealized isotropic scatterer. Thus, the radar cross section of an object is the cross section of an equivalent isotropic scatterer that generates the same scattered power density as the object in the observed direction

$$\sigma = 4\pi R^2 \frac{|\vec{E}^s|^2}{|\vec{E}^i|^2} = 4\pi |S|^2 \quad (4)$$

where $|\vec{E}|^2$ represents the intensity of the electromagnetic field and S is the complex scattering amplitude of the object. The final value of σ is a function of a large number of parameters which are difficult to consider individually. A first set of these parameters are concerned with the imaging system:

- Wave frequency f .

- Wave polarization. This dependence is specially considered later.
- Imaging configuration, that is, incident (θ_i, ϕ_i) and scattering (θ_s, ϕ_s) directions.

A second set of parameters are related with the target itself

- Object geometrical structure.
- Object dielectrical properties.

Then, the radar cross section σ is able to characterize the target being imaged for a particular frequency, and imaging system configuration.

The radar equation, as given by (1), is valid for those cases in which the target of interest is smaller than the radar coverage, that is, a point target. For those targets presenting an extend larger than the radar coverage, we need a different model to represent the target. In these situations, a target is represented as an infinite collection of statistically identical point targets.

Figure 2 presents an scheme of this type of targets.

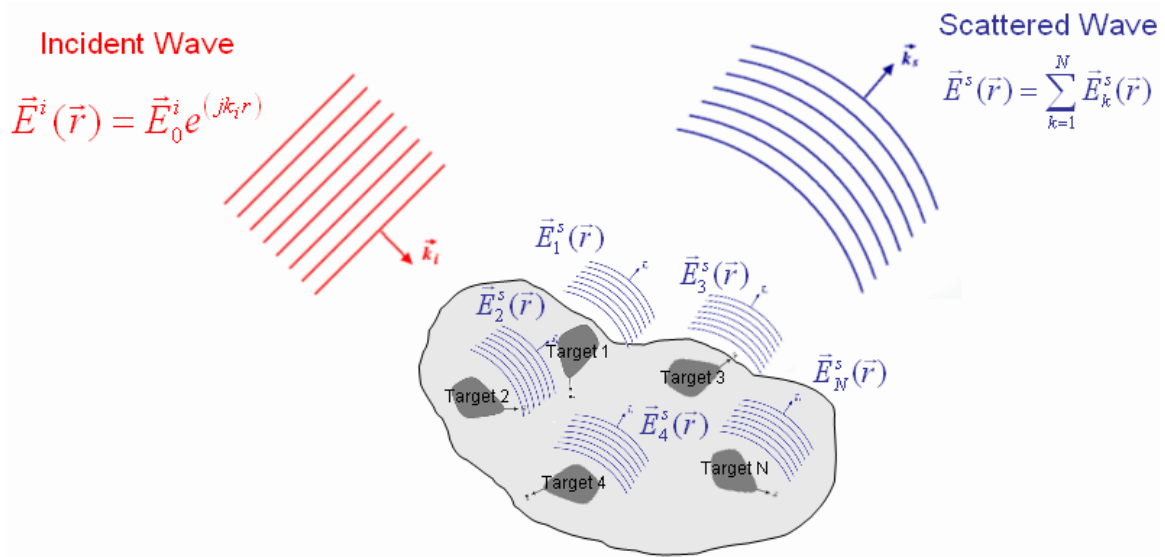


Figure 2 Interaction of an electromagnetic wave and an extended target.

As it can be observed in **Figure 2**, the resulting scattered field \vec{E}^s results from the coherent addition of the scattered waves from every one of the independent targets which model the extended scatterer. In order to express the scattering properties of the extended target independently of its area extend, we consider every elementary target as being described by a differential radar cross section $d\sigma$. In order to separate the effects of the target extend, we consider $d\sigma$ as the product of the averaged radar cross section per unit area σ^0 and the differential area occupied by the target ds . Then, the differential power received by the systems due to an elementary scatterer can be written as

$$dP_r = \frac{P_t G_t}{4\pi R_t^2} \sigma^0 ds \frac{A_r}{4\pi R_r^2} \quad (5)$$

Hence, to find the total power received from the extended target we need to integrate over the illuminated area A_0

$$P_r = \iint_{A_0} \frac{P_i G_t}{4\pi R_t^2} \sigma^0 \frac{A_r}{4\pi R_r^2} ds \quad (6)$$

It must be noted that the radar equation at (1) represents a deterministic problem, whereas (6) considers a statistical problem. Eq. (6) represents the average power returned from the extended target. Hence, the radar cross section per unit area σ^0 , or simply *scattering coefficient*, is the ratio of the statistically averaged scattered power density to the average incident power density over the surface of the sphere of radius R_r .

$$\sigma^0 = \frac{\langle \sigma \rangle}{A_0} = \frac{4\pi R_r^2}{A_0} \frac{\langle |\vec{E}^s|^2 \rangle}{|\vec{E}^i|^2} \quad (7)$$

The scattering coefficient σ^0 is a dimensionless parameter. As in the case of the radar cross section, the scattering coefficient is employed to characterize the scattered being imaged by the radar. This characterization is for a particular frequency f , polarization of the incident and scattered waves and incident (θ_i, ϕ_i) and scattering (θ_s, ϕ_s) directions.

2.1.2 Different Emission-Reception Polarization States

As it has been shown in the previous section, the characterization of a given scatterer by means of the radar cross section σ or the scattering coefficient σ^0 depends also on the polarization of the incident field \vec{E}^i . As one can observe in (4) and (7), these two coefficients are expressed as a function of the intensity of the incident and scattered fields. Consequently, σ and σ^0 shall be only sensible to the polarization of the incident fields through the effects the polarization has over the power of the related electromagnetic waves. Hence, if we denote by p the polarization of the incident field and by q the polarization of the scattered field, we can define the following polarization dependent radar cross section and scattering coefficient respectively

$$\sigma_{qp} = 4\pi R^2 \frac{|\vec{E}_{qp}^s|^2}{|\vec{E}_{qp}^i|^2} = 4\pi |S_{qp}|^2 \quad (8)$$

$$\sigma_{qp}^0 = \frac{\langle \sigma_{qp} \rangle}{A_0} = \frac{4\pi R_r^2}{A_0} \frac{\langle |\vec{E}_{qp}^s|^2 \rangle}{|\vec{E}_{qp}^i|^2} \quad (9)$$

2.1.3 General Case: Scattering Matrix

As it has been shown in the previous two sections, a given target of interest can be characterized by means of the radar cross section or the scattering coefficient depending on the nature of the scatterer itself, see (4) and (7). Additionally, in (8) and (9) it has been shown that these two coefficients depend also on the polarization of the incident and the scattered electromagnetic fields. A closer look to these expressions reveals that these two coefficients

depend on the polarization of the electromagnetic fields only through the power associated with them. Thus, they do not exploit, explicitly, the vectorial nature of polarized electromagnetic waves. Consequently, in order to take advantage of the polarization of the electromagnetic fields, that is, their vectorial nature, the scattering process at the target of interest must be considered as a function of the electromagnetic fields themselves.

In the previous chapter, it was shown that the polarization of a plane, monochromatic, electric field could be represented by the so-called Jones vector. Additionally, a set of two orthogonal Jones vectors form a polarization basis, in which, any polarization state of a given electromagnetic wave can be expressed. Therefore, given the Jones vectors of the incident and the scattered waves, \underline{E}^i and \underline{E}^s respectively, the scattering process occurring at the target of interest is expressed as follows

$$\underline{E}^s = \frac{e^{-jkr}}{r} \left[S_{(\perp, //)} \right] \underline{E}^i = \frac{e^{-jkr}}{r} \begin{bmatrix} S_{\perp\perp} & S_{\perp//} \\ S_{//\perp} & S_{////} \end{bmatrix} \underline{E}^i \quad (10)$$

where the matrix $[S_{(\perp, //)}]$ receives the name of scattering matrix and the entries of this matrix S_{pq} , for $p, q \in (\perp, //)$, are the so-called complex scattering coefficients or complex scattering amplitudes. The diagonal elements of the scattering matrix receive the name of *co-polar* terms, since they relate the same polarization for the incident and the scattered fields. Nevertheless, the off-diagonal elements are known as cross-polar terms as they relate orthogonal polarization states. Finally, the term e^{-jkr}/r takes into account the propagation effects both, in amplitude and phase. It must be taken into account that the relation expressed by (10) is only valid for the far field zone, where the planar wave assumption is considered for the incident and the scattered fields. Considering (8), the elements of the scattering matrix can be related with the radar cross section of a given target as follows

$$|S_{qp}|^2 = \frac{\sigma_{qp}}{4\pi} \quad p, q \in (\perp, //) \quad (11)$$

As it can be deduced from the previous equation, the characterization of a given target by means of the scattering matrix allows the possibility to explore the phase information provided by the phase of complex scattering coefficients, and not only the intensity or amplitude. As one can observe, the polarimetric scattering equation presented at (10) involves the Jones vectors of the incident and the scattered fields, which characterize their polarization properties in a given coordinates systems. In order to be correct, these two Jones vectors must be expressed within the same coordinates reference. As a result, the scattering matrix shall be associated to a particular coordinates system. In (10), we consider the coordinates system centered at the target. Hence, the basis $(\perp, //)$ refers to a plane respect the coordinates systems centered in the target to which the fields and the scattering matrix are referred.

The following example demonstrates the importance of the phase parameters. Let's consider a trihedral and a dihedral. These two targets present the radar cross section coefficients and scattering matrices given in **Figure 3** in the polarization basis formed with the horizontal and vertical polarization states, which are parallel to the \hat{x} and \hat{y} axis, respectively. As it can be concluded from **Figure 3**, the trihedral and the dihedral cannot be differentiated in terms of the radar cross section coefficients, whereas they are seen as different objects if they are analyzed by means of the corresponding scattering matrices. The conclusion which can be

extracted at this point is that polarimetry opens the door to consider phase measurements to characterize the targets.

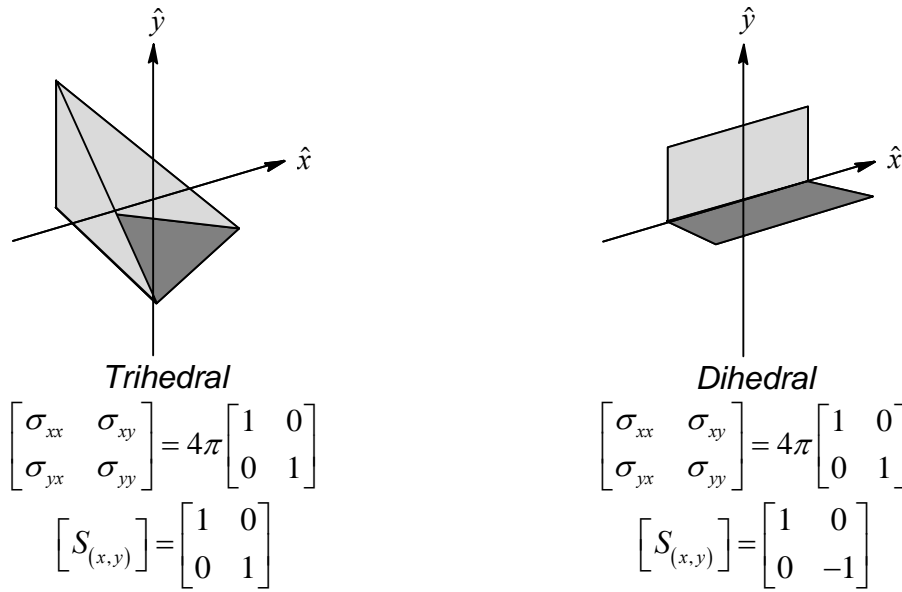


Figure 3 Trihedral and dihedral polarimetric characterization.

Since the scattering matrix $[S_{(\perp, //)}]$ is employed to characterize a given target, it can be parameterized as follows

$$\begin{bmatrix} S_{(\perp, //)} \end{bmatrix} = \frac{e^{-jkr}}{r} \begin{bmatrix} |S_{\perp\perp}| e^{j\varphi_{\perp\perp}} & |S_{\perp//}| e^{j\varphi_{\perp//}} \\ |S_{//\perp}| e^{j\varphi_{//\perp}} & |S_{////}| e^{j\varphi_{////}} \end{bmatrix} = \underbrace{\frac{e^{-jkr} e^{j\varphi_{\perp\perp}}}{r}}_{\text{Absolute Phase Term}} \underbrace{\begin{bmatrix} |S_{\perp\perp}| & |S_{\perp//}| e^{j(\varphi_{\perp//} - \varphi_{\perp\perp})} \\ |S_{//\perp}| e^{j(\varphi_{//\perp} - \varphi_{\perp\perp})} & |S_{////}| e^{j(\varphi_{////} - \varphi_{\perp\perp})} \end{bmatrix}}_{\text{Relative Scattering Matrix}} \quad (12)$$

The absolute phase term in (12) is not considered as an independent parameter since it presents an arbitrary value due to its dependence on the distance between the radar and the target. Consequently, it is assumed that the scattering matrix can be parameterized by 7 independent parameters: the amplitudes $\{|S_{\perp\perp}|, |S_{\perp//}|, |S_{//\perp}|, |S_{////}|\}$ and the relative phases $\{(\varphi_{\perp//} - \varphi_{\perp\perp}), (\varphi_{//\perp} - \varphi_{\perp\perp}), (\varphi_{////} - \varphi_{\perp\perp})\}$. As a conclusion, a given target of interest is determined by 7 independent parameters in the most general case and an absolute value.

It is important, at this point, to analyze some particular aspects about the definition of the matrix $[S_{(\perp, //)}]$ and the relation about the different coordinates systems which can be defined to describe the scattering process characterized in (10).

As it was already highlighted in the previous two sections of this chapter, the radar cross section and the scattering coefficients depend on the direction of the incident and the scattered waves. When considering the matrix $[S_{(\perp, //)}]$, the analysis of this dependence is of extreme importance since it also involves the definition of the polarization of the incident and the scattered fields. Since (10) considers the polarized electromagnetic waves themselves, it is mandatory to assume a frame in which the polarization is defined. There exist two principal conventions concerning the framework where the polarimetric scattering process is considered: *Forward Scatter Alignment* (FSA) and *Backscatter Alignment* (BSA), see **Figure 4**. In both cases, the electric fields of the incident and the scattered waves are expressed in local coordinates systems centered on the transmitting and receiving antennas, respectively.

All coordinate systems are defined in terms of a global coordinate system centered inside the target of interest.

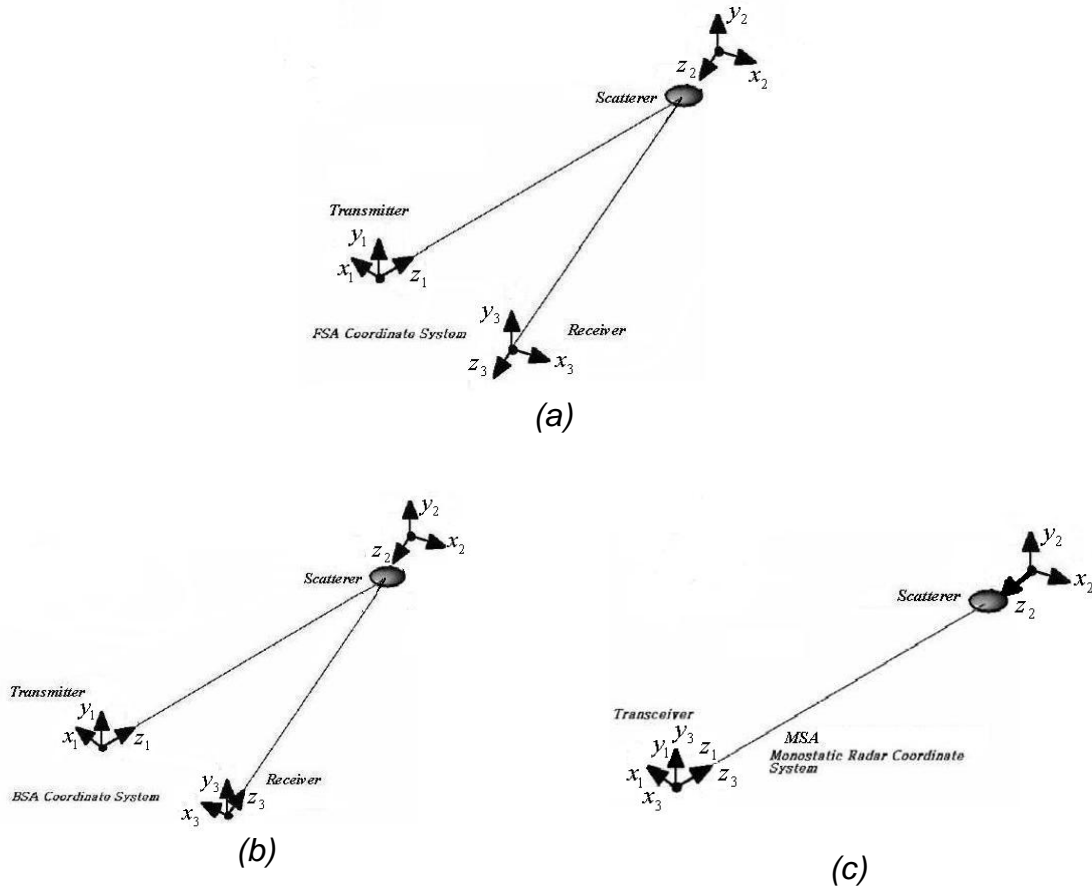


Figure 4 Reference frameworks: (a) FSA, (b) Bistatic BSA and (c) Monostatic BSA.

The FSA convention, see **Figure 4a**, also called *wave-oriented* since it is defined relative to the propagating wave, is normally considered in bistatic problems, that is, in those configurations in which the transmitter and the receiver are not located at the same spatial position.

The bistatic BSA convention framework, see **Figure 4b**, is defined, on the contrary, respect to the radar antennas in accordance with the IEEE standard. The advantage of the BSA convention is that for a monostatic configuration, also called *backscattering* configuration, that is, when the transmitting and receiving antennas are collocated, the coordinated systems of the two antennas coincide, see **Figure 4c**. This configuration is preferred in the radar polarimetry community. In the monostatic case, the scattering matrix in the FSA convention, $[S_{(\perp, //)}]_{FSA}$, can be related with the same matrix referenced to the monostatic BSA convention $[S_{(\perp, //)}]_{BSA}$ as follows

$$[S_{(\perp, //)}]_{BSA} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} [S_{(\perp, //)}]_{FSA} \quad (13)$$

As it has been mentioned previously, in the radar polarimetry community, the monostatic BSA convention (backscattering) is considered as the framework to characterize the scattering

process. The reason to select this configuration is due to fact that the majority of the existing polarimetric radar systems operate with the same antenna for transmission and reception. One important property of this configuration, for reciprocal targets, is *reciprocity*, which states that

$$[S_{\perp//}]_{BSA} = [S_{//\perp}]_{BSA} \quad (14)$$

$$[S_{\perp//}]_{FSA} = -[S_{//\perp}]_{FSA} \quad (15)$$

Then, the formalization of the scattering process given by (10), in the monostatic case under the BSA convention, reduces to

$$\underline{E}^S = \frac{e^{-jkr}}{r} [S_{(\perp, //)}] \underline{E}^i = \frac{e^{-jkr}}{r} \begin{bmatrix} S_{\perp\perp} & S_{\perp//} \\ S_{\perp//} & S_{//\perp} \end{bmatrix} \underline{E}^i \quad (16)$$

In the same sense, equation (12) takes the form

$$[S_{(\perp, //)}] = \frac{e^{-jkr}}{r} \begin{bmatrix} |S_{\perp\perp}| e^{j\varphi_{\perp\perp}} & |S_{\perp//}| e^{j\varphi_{\perp//}} \\ |S_{\perp//}| e^{j\varphi_{\perp//}} & |S_{//\perp}| e^{j\varphi_{//\perp}} \end{bmatrix} = \underbrace{\frac{e^{-jkr} e^{j\varphi_{\perp\perp}}}{r}}_{\text{Absolute Phase Term}} \underbrace{\begin{bmatrix} |S_{\perp\perp}| & |S_{\perp//}| e^{j(\varphi_{\perp//} - \varphi_{\perp\perp})} \\ |S_{\perp//}| e^{j(\varphi_{\perp//} - \varphi_{\perp\perp})} & |S_{//\perp}| e^{j(\varphi_{//\perp} - \varphi_{\perp\perp})} \end{bmatrix}}_{\text{Relative Scattering Matrix}} \quad (17)$$

The main consequence of (17) is that in the backscattering direction, a given target is no longer characterized by 7 independent parameters, but by 5. These are: the amplitudes $\{|S_{\perp\perp}|, |S_{\perp//}|, |S_{//\perp}|\}$ and the relative phases $\{(\varphi_{\perp//} - \varphi_{\perp\perp}), (\varphi_{//\perp} - \varphi_{\perp\perp})\}$ and one additional absolute phase.

2.2 Additional Information

2.2.1 Span: Joint Intensity Information

As it was indicated at the beginning of Section 2.1.1, a central parameter when considering the scattering process occurring at a given target consists of the scattered power. For single polarization systems, the scattered power is determined by means of the radar cross section or the scattering coefficient. Nevertheless, a polarimetric radar has to be considered as a multi channel system. Consequently, in order to determine the scattered power, it is necessary to consider all the data channels, that is, all the elements of the scattering matrix. The total scattered power, in the case of a polarimetric radar system is know as span, being defined in the most general case as

$$SPAN([S_{(\perp, //)}]) = \text{trace}([S_{(\perp, //)}][S_{(\perp, //)}]^{T*}) = |S_{\perp\perp}|^2 + |S_{\perp//}|^2 + |S_{//\perp}|^2 + |S_{//\perp}|^2 \quad (18)$$

where $\text{trace}(\cdot)$ represents the trace of a matrix, consisting of the addition of the elements of the principal diagonal. In the backscattering case, due to the reciprocity theorem, the span reduces to

$$SPAN([S_{(\perp, //)}]) = |S_{\perp\perp}|^2 + 2|S_{\perp//}|^2 + |S_{//\perp}|^2 \quad (19)$$

The main property of the span is that it is *polarimetrically invariable*, that is, it does not depend on the polarization basis employed to describe the polarization of the electromagnetic waves.

Now, if we consider the definition (11) into (19) we get

$$SPAN\left(\left[S_{(\perp,\parallel)}\right]\right) = 4\pi\left(\sigma_{\perp\perp} + 2\sigma_{\perp\parallel} + \sigma_{\parallel\parallel}\right) \quad (20)$$

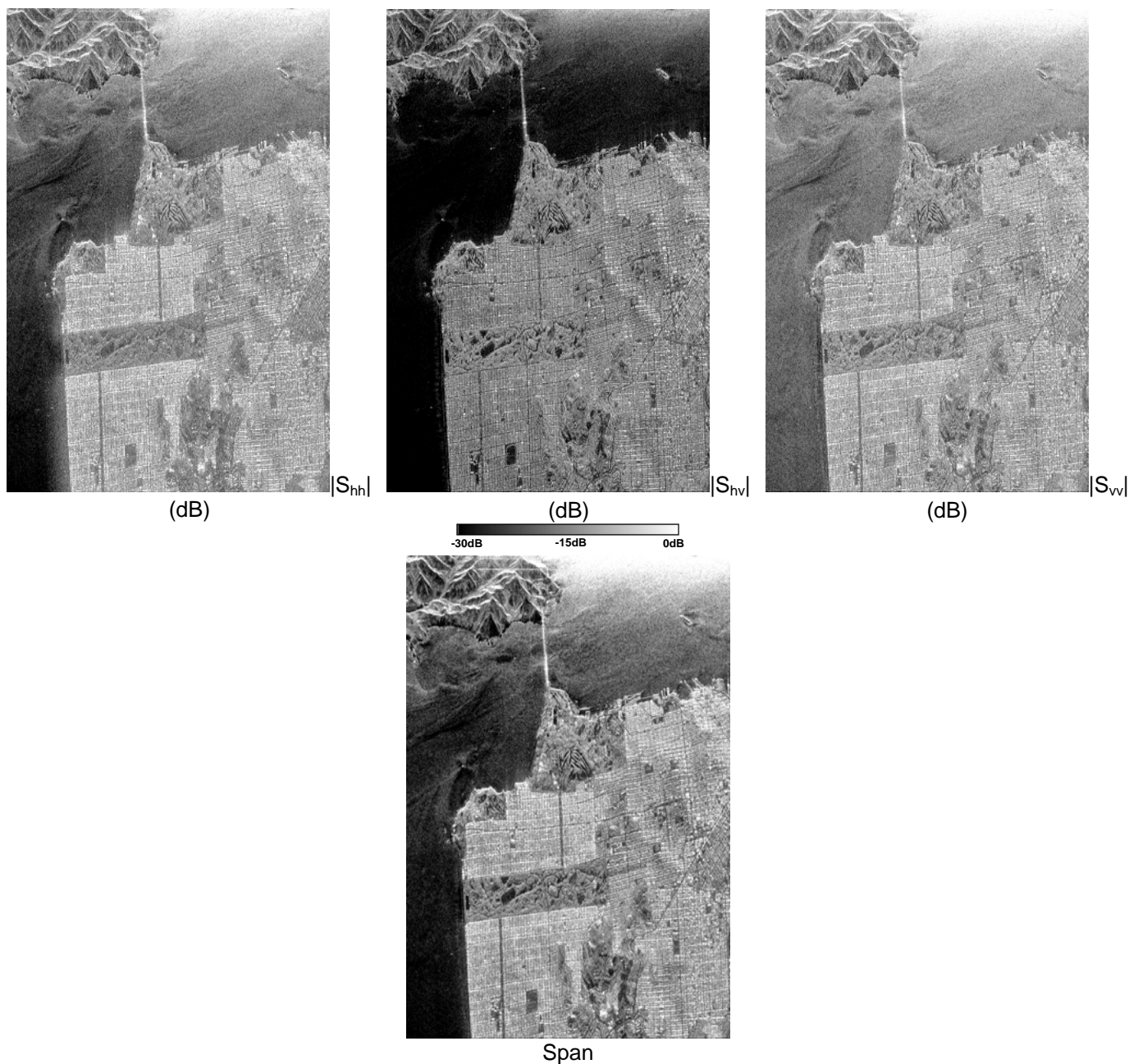


Figure 5 Intensities of the elements of the scattering matrix measured in the basis (h, v) and the resulting span.

Therefore, the span presents the same limitations as the radar cross section in order to represent the polarimetric information contained in the scattering matrix, that is, the important

information provided by the measurement of the relative phases is completely lost. Additionally, since it is polarimetrically invariable, it does not contain polarimetric information at all.

One of the main applications of the span is to present the multidimensional information provided by the scattering matrix in a single image. But, as shown in the previous paragraph, this image does not contain any polarimetric information at all. **Figure 5** presents an example of the span image.

2.2.2 Lexicographic Color-Coded Representation

In the previous section, we showed that it was possible to present part of the information contained in the scattering matrix $[S_{(\perp, //)}]$ in a single image by means of the span of the scattering matrix, but at the expense to loose all the polarimetric information. In what it follows, we introduce two alternatives to represent part of the multidimensional information provided by the scattering matrix in a single image. These two alternatives are based on the decomposition RBG of color images.

2.2.2.1 RBG color coding

The RGB color model is an additive color model in which red, green, and blue light are combined in various ways to create other colors. The very idea for the model itself and the abbreviation "RGB" come from the three primary colors in additive light models. The concept to retain is that we can create any color of the spectra by combining the three primary colors: red, green and blue light, that is, color can be considered as a three dimensional space. **Figure 6** presents the color space, coded in the RGB base.

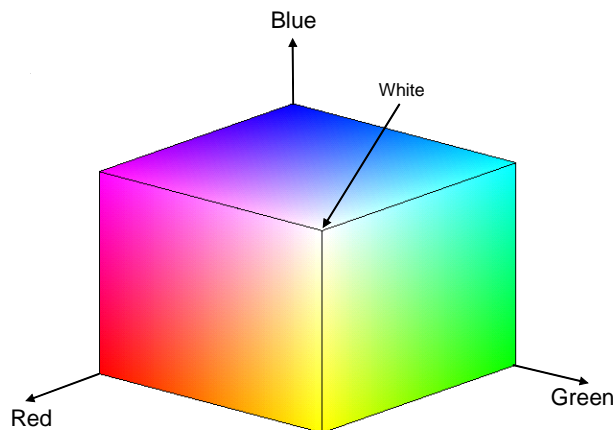


Figure 6 RBG color cube.

Consequently, three different parameters can be represented in a single color image by coding each initial parameters as one color of the RGB space. Therefore, the color in the resulting image can be interpreted in terms of the three selected parameters.

2.2.2.2 Polarimetric information coded as a color

The color representation presented above is now considered to represent the polarimetric information provided by the scattering matrix $[S_{(\perp, //)}]$. The lexicographic color-coded

representation considers the intensity of the elements of $[S_{(\perp, //)}]$, in the backscattering direction, as the dimensions of the color space. A possible codification could be

$$\begin{aligned} |S_{\perp\perp}|^2 &\rightarrow \text{Red} \\ 2|S_{\perp//}|^2 &\rightarrow \text{Green} \\ |S_{//}|^2 &\rightarrow \text{Blue} \end{aligned} \quad (21)$$

despite any other codification is also valuable. Again, it must be mention that, since we are considering the intensity of the different elements of the scattering matrix, this codification is not able to reflect the information that can be provided by the relative phases of elements.

2.2.3 Importance of Relative Phase Terms: Pauli Color-Coded Representation

In the following we present a color-coded representation of the information provided by the scattering matrix which takes into account the information provided by the relatives phases of the different entries of the matrix. Any 2×2 symmetric matrix can be parameterized as a function of three parameters $\{a, b, c\}$ as follows

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} = \begin{bmatrix} a+b & c \\ c & a-b \end{bmatrix} \quad (22)$$

If we now consider the representation of the scattering matrix in the backscattering case presented in (16), we can obtain the following equivalences

$$\begin{aligned} a &= \frac{S_{\perp\perp} + S_{//}}{\sqrt{2}} \\ b &= \frac{S_{\perp\perp} - S_{//}}{\sqrt{2}} \\ c &= \sqrt{2}S_{\perp//} \end{aligned} \quad (23)$$

The Pauli color-coded representation considers the codification in color of the intensity of the parameters in (23) as follows

$$\begin{aligned} |b|^2 &\rightarrow \text{Red} \\ |c|^2 &\rightarrow \text{Green} \\ |a|^2 &\rightarrow \text{Blue} \end{aligned} \quad (24)$$

Finally, **Figure 8a** presents an example or the Pauli Color coded representation of the image channels presented in **Figure 5**.

2.3 Polarization Basis and Polarimetric Signature

2.3.1 Acquisition According to Orthogonal Jones Vectors: Polarization Basis

As it was demonstrated in the preceding chapter, the Jones vector corresponding to a given electromagnetic field can be expressed in an orthonormal basis ($\perp, //$) as

$$\underline{E} = E_{\perp} \hat{\perp} + E_{//} \hat{//} \quad (25)$$

where the unitary vectors $\hat{\perp}$ and $\hat{//}$ form the so-called polarization basis. If now, we consider the polarimetric scattering equation, see (10), the Jones vectors corresponding to the incident and the scattered waves, \underline{E}^i and \underline{E}^s respectively, are expressed in the orthonormal basis ($\perp, //$). Consequently, the scattering matrix $[S_{(\perp, //)}]$ specifies how the target transforms the polarization of the incident field to the scattered field when both are expressed in the basis ($\perp, //$). As a result, it is said that the scattering matrix $[S_{(\perp, //)}]$ has been acquired with respect to the polarization basis ($\perp, //$).

2.3.2 Acquisition in ONE Polarization Basis: Representation in ANY Polarization Basis

The scattering properties of a given target, as demonstrated, are contained within the scattering matrix $[S_{(\perp, //)}]$, which, as shown previously, is measured in the particular polarization basis ($\perp, //$). Since, there exist an infinite number of orthonormal polarization bases, the question raising at this point is that whether it is possible or not to infer the polarimetric properties of the given target in any polarization basis from the response measured at a particular basis, for instance ($\perp, //$). This question presents an affirmative answer. The possibility to *synthesize* any polarimetric response of a given target from its measurement in a particular orthonormal basis represents the most important property of polarimetric systems in comparison with single-polarization systems. The most important consequence of this process is that the amount of information about a given scatterer can be increased, allowing a better characterization and study. This polarization synthesis process is based on the concept of change of polarization basis presented in the previous chapter. In what it follows, we shall consider the polarization synthesis process in the backscattering direction or monostatic configuration of a radar system.

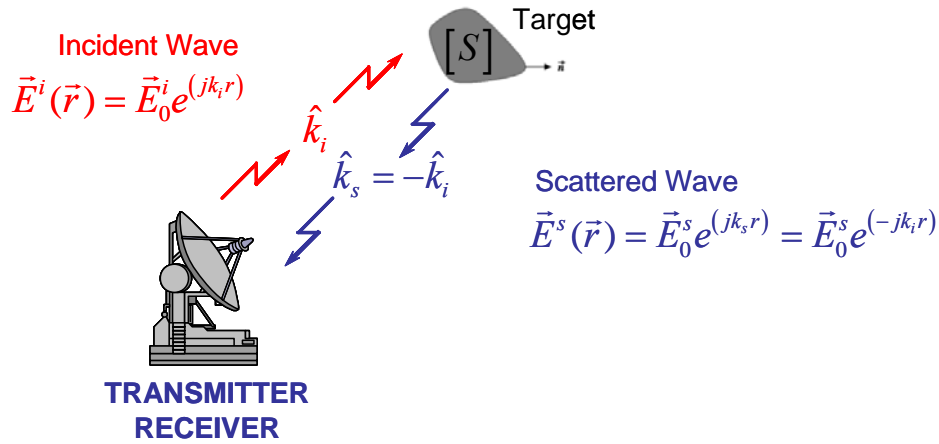


Figure 7 Polarization synthesis process in the backscattering direction.

Before to describe the polarization synthesis process in the backscattering direction, it is necessary to analyze the scattering process given by (10) with respect to the direction of propagation of the incident and the scattered fields. As observed in **Figure 7**, the incident field propagates in the direction given by the unitary vector \hat{k}_i , whereas the scattered one propagates in the opposite direction, given by \hat{k}_s . Consequently, this difference in the propagation direction must be taken into account when defining the polarization state of the wave. Given a Jones vector propagating in the direction \hat{k} , the Jones vector of the same wave, but which propagates in the direction $-\hat{k}$ is obtained as

$$\hat{k} \rightarrow -\hat{k} \quad \underline{E}(\hat{k}) = \underline{E}^*(-\hat{k}) \quad (26)$$

From now, we shall not consider the scattering matrix referred to the coordinates system centered at the target, but in the axis coordinate centered in the transmitting/receiving system. Consider a polarimetric radar system as depicted by **Figure 7**, which transmits the electromagnetic waves in the following orthonormal basis (A, A_\perp) . In this particular basis, the incident and scattered fields are related by the scattering matrix as follows

$$\underline{E}_{(A, A_\perp)}^s = \begin{bmatrix} S_{AA} & S_{AA_\perp} \\ S_{AA_\perp} & S_{A_\perp A_\perp} \end{bmatrix} \underline{E}_{(A, A_\perp)}^i = \left[S_{(A, A_\perp)} \right] \underline{E}_{(A, A_\perp)}^i \quad (27)$$

As shown in the previous chapter, given the Jones vector measured in a particular basis, for instance (A, A_\perp) , it is possible to derive it in any other polarization basis (B, B_\perp) by means of the following simple mathematical transformation

$$\underline{E}_{(B, B_\perp)} = \left[U_{(A, A_\perp) \rightarrow (B, B_\perp)} \right] \underline{E}_{(A, A_\perp)} \quad (28)$$

Then, if we consider (28) for the incident Jones vector in (27), we get

$$\underline{E}_{(B, B_\perp)}^i = \left[U_{(A, A_\perp) \rightarrow (B, B_\perp)} \right] \underline{E}_{(A, A_\perp)}^i \quad (29)$$

$$\underline{E}_{(A,A_{\perp})}^i = \left[U_{(B,B_{\perp}) \mapsto (A,A_{\perp})} \right] \underline{E}_{(B,B_{\perp})}^i \quad (30)$$

In order to apply the transformation basis procedure to the scattered field $\underline{E}_{(A,A_{\perp})}^s$ we need to consider that it propagates in the opposite direction as the incident field $\underline{E}_{(A,A_{\perp})}^i$. Hence, in order to consider both fields in the same frame of reference, that is, their polarizations are referred to the same coordinate system we need to consider (26). As a result, the transformation basis procedure applies to the scattered field as follows

$$\underline{E}_{(B,B_{\perp})}^s = \left[U_{(A,A_{\perp}) \mapsto (B,B_{\perp})} \right]^* \underline{E}_{(A,A_{\perp})}^s \quad (31)$$

$$\underline{E}_{(A,A_{\perp})}^s = \left[U_{(B,B_{\perp}) \mapsto (A,A_{\perp})} \right]^* \underline{E}_{(B,B_{\perp})}^s \quad (32)$$

Now, introducing the results of (30) and (32) into (27) we get

$$\left[U_{(B,B_{\perp}) \mapsto (A,A_{\perp})} \right]^* \underline{E}_{(B,B_{\perp})}^s = \left[S_{(A,A_{\perp})} \right] \left[U_{(B,B_{\perp}) \mapsto (A,A_{\perp})} \right] \underline{E}_{(B,B_{\perp})}^i \quad (33)$$

$$\underline{E}_{(B,B_{\perp})}^s = \left(\left[U_{(B,B_{\perp}) \mapsto (A,A_{\perp})} \right]^* \right)^{-1} \left[S_{(A,A_{\perp})} \right] \left[U_{(B,B_{\perp}) \mapsto (A,A_{\perp})} \right] \underline{E}_{(B,B_{\perp})}^i \quad (34)$$

Since the transformation matrix $\left[U_{(A,A_{\perp}) \mapsto (B,B_{\perp})} \right]$ is unitary, i.e., $[U]^{-1} = [U]^{*T}$, we get



(a) $|S_{hh} + S_{vv}|$ $|S_{hh} - S_{vv}|$ $|S_{hv}|$

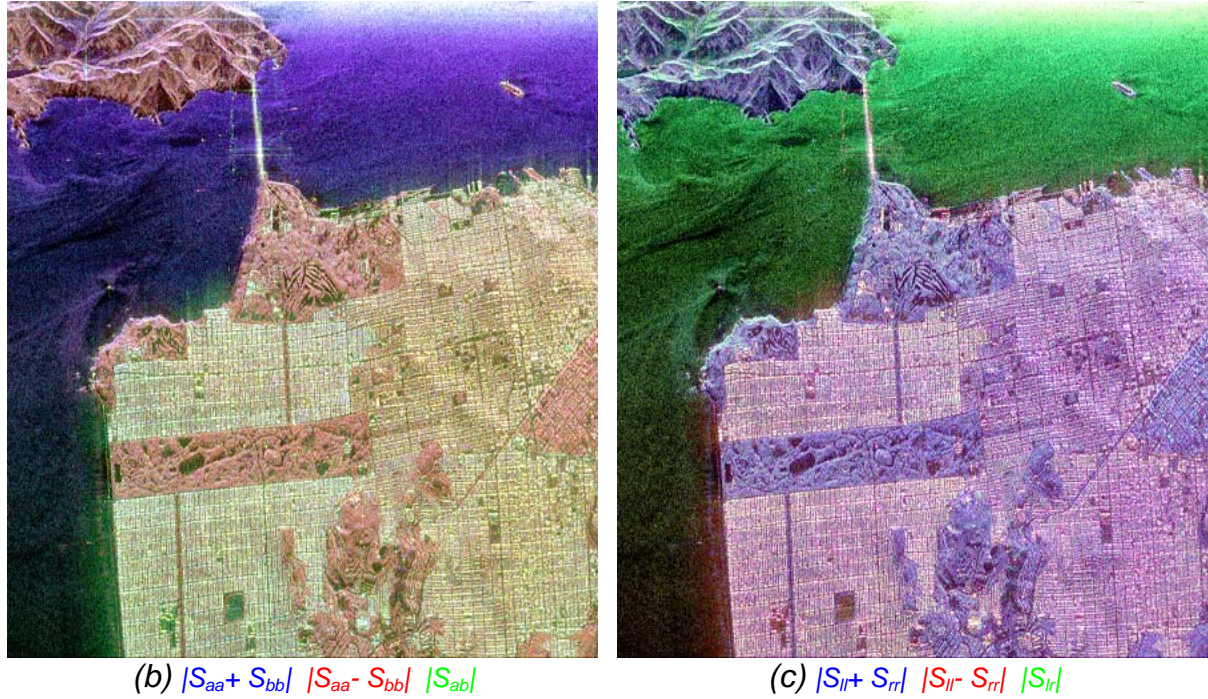


Figure 8 Polarization synthesis. (a) Measured response at the linear polarization basis (h,v). (b) Synthesis of the response in the linear basis rotated 45 degree (a,b). (c) Synthesis in the circular polarization basis (l,r).

$$\underline{E}_{(B,B_{\perp})}^s = \left[U_{(B,B_{\perp}) \rightarrow (A,A_{\perp})} \right]^T \left[S_{(A,A_{\perp})} \right] \left[U_{(B,B_{\perp}) \rightarrow (A,A_{\perp})} \right] \underline{E}_{(B,B_{\perp})}^i \quad (35)$$

In the previous equation, we can clearly identify, then,

$$\left[S_{(B,B_{\perp})} \right] = \left[U_{(B,B_{\perp}) \rightarrow (A,A_{\perp})} \right]^T \left[S_{(A,A_{\perp})} \right] \left[U_{(B,B_{\perp}) \rightarrow (A,A_{\perp})} \right] \quad (36)$$

The transformation expressed in (36) receives the name of *con-similarity transformation*. This transformation allows to synthesize the scattering matrix in an arbitrary basis (B, B_{\perp}), from its measure in the basis (A, A_{\perp}).

If instead of an arbitrary basis, we consider the linear orthogonal polarization basis (\hat{x}, \hat{y}), the transformation matrix from (\hat{x}, \hat{y}) to an arbitrary elliptical basis can be parameterized as

$$\begin{aligned} \left[U_{(\hat{x}, \hat{y}) \rightarrow (A_{\perp}, A_{//})} \right] &= \left[U(\phi) \right] \left[U(\tau) \right] \left[U(\alpha) \right] \\ &= \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \tau & j \sin \tau \\ j \sin \tau & \cos \tau \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \end{aligned} \quad (37)$$

Figure 8 presents an example of the application of the con-similarity transformation given at (36) to synthesize the polarimetric response at different polarization basis. The polarimetric information is represented by means of the Pauli color-coded representation. The original polarimetric set, presented in **Figure 8a** is obtained in the linear polarization basis (h,v), where h stands for the horizontal polarization and v for the vertical polarization. Using (37) the response to two different polarization basis is synthesized. **Figure 8b** presents the response to the orthonormal basis (a,b) where a indicates the linear polarization at 45 degrees

and b the linear polarization at -45 degrees. Finally, **Figure 8c** shows the response to the circular polarization basis (l,r) , where l refers to the left circular polarization and r to the right circular polarization.

2.3.3 Complete Polarimetric Characterization of Each Pixel: Polarization Signature

In the previous section, we have employed the powerful con-similarity transformation to synthesize the polarimetric response just in two alternative polarization bases: the 45 degrees rotated linear basis (a, b) and the circular polarization basis (l, r) . Nevertheless, this section aims to exploit all the information which can be extracted from this transformation through the exploration of the space of all possible polarization bases. The objective of this process is to present a new way to characterize a given scatterer.

Figure 9 presents the scheme of a general polarimetric radar employed to measure a given target, characterized by a particular scattering matrix. In this scheme, the Jones vector \underline{A} refers to the polarization emitted by the transmitting system and \underline{B} indicates the Jones vector containing the polarization with respect to which the receiving antenna receives the scattered field \underline{E} . In this context, the power received at the receiving system is obtained by means of

$$P = \alpha |\underline{B}^T \underline{E}|^2 \quad (38)$$

where α contains all those terms which do not depend on the polarization of the antennas \underline{A} and \underline{B} . Using the characterization of the target by means of the scattering matrix, it is possible to rewrite (38) as follows

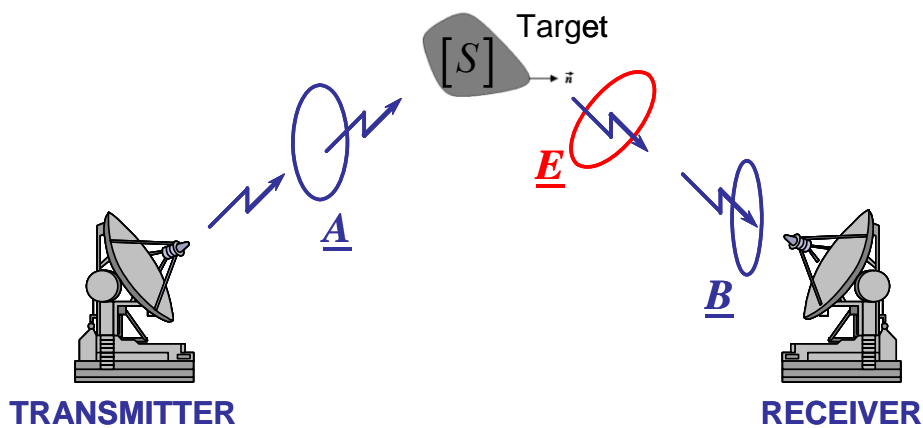


Figure 9 General polarimetric system configuration.

$$P_A^B = \alpha |\underline{B}^T [S] \underline{A}|^2 \quad (39)$$

In the frame of (39), two different power measurements are defined:

- The *co-polarized* power. In this configuration, we assume that the transmitting and receiving antennas are characterized by the same polarization state. Hence, (39) turns out to be

$$P_{CO} = \alpha \left| \underline{A}^T [S] \underline{A} \right|^2 \quad (40)$$

- The *cross-polarized* power. In this case, it is assumed that the receiver antenna receives with the orthogonal polarization of the transmitting system. Thus, we write (39) as

$$P_X = \alpha \left| \underline{A}_\perp^T [S] \underline{A} \right|^2 \quad (41)$$

The co- and cross-polarized powers can be generated synthetically, since, given the linear polarization basis (\hat{x}, \hat{y}) , we can write an arbitrary polarization state

$$\underline{A} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \tau & j \sin \tau \\ j \sin \tau & \cos \tau \end{bmatrix} \hat{x} \quad (42)$$

and

$$\underline{A} = \begin{bmatrix} \cos\left(\phi + \frac{\pi}{2}\right) & -\sin\left(\phi + \frac{\pi}{2}\right) \\ \sin\left(\phi + \frac{\pi}{2}\right) & \cos\left(\phi + \frac{\pi}{2}\right) \end{bmatrix} \begin{bmatrix} \cos(-\tau) & j \sin(-\tau) \\ j \sin(-\tau) & \cos(-\tau) \end{bmatrix} \hat{x} \quad (43)$$

Consequently, we can obtain the co- and cross-polarized powers, P_{CO} and P_X , as a function of the angles ϕ and τ , that is, the ellipse orientation and the ellipse aperture which define the polarization ellipse of an electromagnetic wave. We have not considered here the phase term α .

Section 2.5 presents the polarization signatures for the canonical scattering mechanisms.

2.4 Optimal Polarization States and The Poincaré Sphere

2.4.1 Optimal Polarization States: Deduction Oriented Parameterization

As it has been shown in the previous section of this chapter, the received power at the receiver system varies according to the polarization of the transmitted and incident waves, and on the characteristics of the target under study through its scattering matrix $[S]$, (39). In the case of the polarization signature, we particularized the study of the received power to two special cases. On the one hand, we have considered the situation in which the transmitting and receiving systems employ the same polarization. On the other hand, the case in which both systems use orthogonal polarizations has been analyzed. The polarization signatures for the co-polar and cross-polar configurations, given by (40) and (41) respectively, explore all the

space of possible polarizations. Nevertheless, the characterization of a particular scatterer can be also performed by studying the so called *characteristic polarization states*. These states are defined as such wave polarization states giving as a result maximum or minimum received powers.

Given a particular polarization state for an electromagnetic wave, the corresponding Jones vector can be represented in terms of the polarization ratio ρ as presented in the previous chapter. Then for the Jones vector \underline{A}

$$\underline{A} = \frac{e^{-j\alpha}}{\sqrt{1+|\rho|^2}} \begin{bmatrix} 1 \\ \rho \end{bmatrix} \quad (44)$$

The Jones vector of the orthogonal state \underline{A}_\perp is

$$\underline{A}_\perp = \frac{e^{+j\alpha}}{\sqrt{1+|\rho|^2}} \begin{bmatrix} -\rho^* \\ 1 \end{bmatrix} \quad (45)$$

If now, we consider the scattering matrix as

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} \quad (46)$$

the co-polar P_{CO} and cross-polar P_X can be respectively written as follows

$$P_{CO} = \alpha \left| S_{XX} + 2\rho S_{XY} + \rho^2 S_{YY} \right|^2 \quad (47)$$

$$P_X = \alpha \left| \rho^* S_{XX} - (1 - |\rho|^2) S_{XY} + \rho S_{YY} \right|^2 \quad (48)$$

Then, in order to derive the characteristic polarization states, we need to calculate the following derivatives

$$\frac{\partial P_{CO}}{\partial \rho} = 0 \quad (49)$$

$$\frac{\partial P_X}{\partial \rho} = 0 \quad (50)$$

As one can observe, (47) and (48) consist of bilinear forms. Consequently, the processes presented at (49) and (50) to derive the characteristic polarization states shall present two solutions. The next two sections analyze the characteristics states resulting from (49) and (50).

2.4.1.1 Characteristics Polarization States in the Co-polar Configuration

The co-polar power P_{CO} presents two polarizations states resulting in maximum received power. These two states are called *COPOL MAX* and are represented by the pair of polarization states $(\underline{K}, \underline{L})$. Then,

$$P_K^K \Rightarrow \text{Global Maximum} \quad (51)$$

$$P_L^L \Rightarrow \text{Local Maximum} \quad (52)$$

The COPOL MAX polarizations states are orthogonal.

Additionally, the co-polar power P_{CO} has two characteristic polarization states for which the received power is zero. This pair of polarization states are named *COPOL NULLS* and are represented by $(\underline{Q}_1, \underline{Q}_2)$. These polarization states give as a result

$$P_{\underline{Q}_1}^{Q_1} = 0 \quad (53)$$

$$P_{\underline{Q}_2}^{Q_2} = 0 \quad (54)$$

The main characteristic of these states is that they are not mutually orthogonal.

2.4.1.2 Characteristics Polarization States in the Cross-polar Configuration

In the case of the cross-polar power P_X , it is possible to find three pairs of orthogonal polarization states which result into characteristic polarization states.

The first pair of polarization states results into maximum received power at the receiver system. This pair of polarization states receives the name of *XPOL MAX* and are represented by the pair $(\underline{C}_1, \underline{C}_2)$. These states result into

$$P_{\underline{C}_1}^{C_1} \Rightarrow \text{Global Maximum} \quad (55)$$

$$P_{\underline{C}_2}^{C_2} \Rightarrow \text{Local Maximum} \quad (56)$$

The states $(\underline{C}_1, \underline{C}_2)$ are mutually orthogonal.

The second pair of polarization states gives null received power. This set of polarization states is known as *XPOL NULL* and are represented by $(\underline{X}_1, \underline{X}_2)$, resulting into

$$P_{\underline{X}_1}^{X_1} = 0 \quad (57)$$

$$P_{\underline{X}_2}^{X_2} = 0 \quad (58)$$

As first established by Kennaugh, the *XPOL NULL* and the *COPOL MAX* represent the same pair of polarization states. Consequently, $(\underline{X}_1, \underline{X}_2)$ consist also in orthogonal polarization states.

Finally, in the case of the cross-polar power, it is possible to define a third pair of polarizations states which can be taken as characteristics. These polarization states result into a minimum received power and are known as *XPOL SADDLE*. They are represented by $(\underline{D}_1, \underline{D}_2)$ and

$$P_{\underline{D}_1}^{D_1} \Rightarrow \text{Global Minimum} \quad (59)$$

$$P_{\underline{D}_2}^{D_2} \Rightarrow \text{Local Minimum} \quad (60)$$

Again, $(\underline{D}_1, \underline{D}_2)$ consist of mutually orthogonal polarization states.

2.4.2 Representation on The Poincare Sphere

In the former chapter, we presented the representation of a given polarization state into the so-called Poincare Sphere. In the following, we are going to make use of this representation to show that the characteristic polarization states pairs given previously present a particular shape into the Poincare sphere. **Figure 10** presents the representation of the characteristic polarization states within the Poincare sphere.

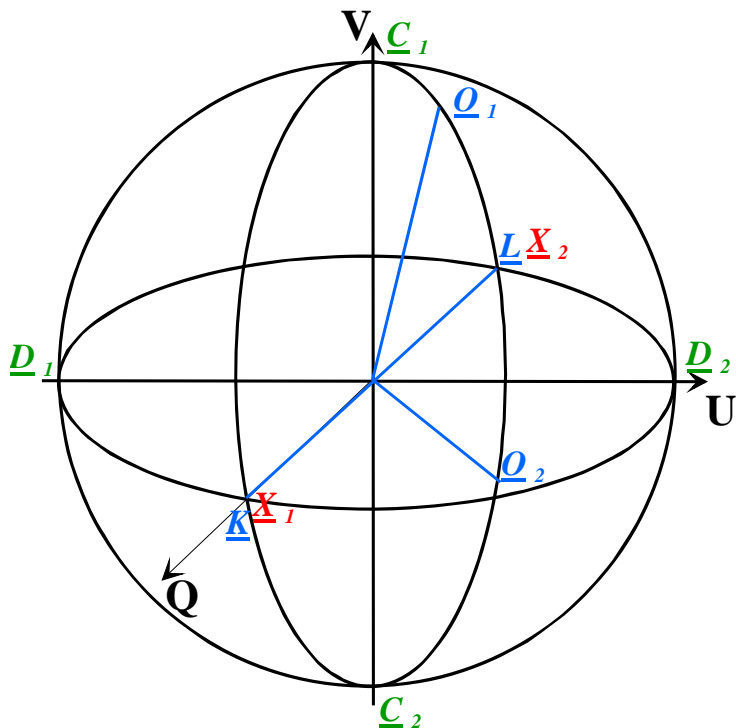


Figure 10 Representation in the Poincare Sphere of the five pairs of characteristic polarization states.

In the previous figure, it can be clearly observed that all the pairs except (Q_1, Q_2) represent pairs of orthogonal polarization states since they consist of antipodal points of the Poincare Sphere. Additionally, it is possible to recognize that the orthogonal pairs of polarization states *XPOL NULL* (equal to the *COPOL MAX*) and *XPOL MAX* define a plane within the Poincare sphere which also contains the non-orthogonal pair of *COPOL NULLS*.

2.4.3 Polarization Fork

In the previous section, we have considered the representation of the five pairs of characteristic polarization states within the Poincare Sphere. Huynen introduced the concept of *Polarization Fork* by considering only the representation of the pairs (K, L) or (X_1, X_2) and the non-orthogonal set (Q_1, Q_2) . **Figure 11** gives the representation of the polarization fork. In order to better understand this concept it is helpful to consider the parameterization of the scattering matrix proposed by Hyunen.

Given an arbitrary scattering matrix, we want to find the rotation matrix $[U]$, in the sense of (37), which gives as a result a diagonal scattering matrix. In general, the diagonalization of a matrix is performed by the standard eigen-decomposition of matrices. Nevertheless, in the backscattering case under the BSA convention, we have to be aware that we are dealing with electromagnetic waves traveling in opposite directions. Consequently, the diagonalization of the scattering matrix must be done according to the con-similarity transformation, as presented in Section 2.3.2. In this particular situation, the diagonalization of the scattering matrix is done with the pseudo eigen-decomposition, that is,

$$[S]\underline{X} = \lambda \underline{X}^* \quad (61)$$

where λ refers to the pseudo-eigenvalues of $[S]$ and \underline{X} to the corresponding pseudo-eigenvectors. Since the scattering matrix consists of a 2×2 complex symmetric matrix in the backscattering case under the BSA convention, (61) presents two solutions. As demonstrated by Huynen, the pseudo-eigenvectors of the scattering matrix correspond to the *XPOL NULL*, i.e., $(\underline{X}_1, \underline{X}_2)$. Since these two polarization states are orthogonal, it is only necessary to specify one of them. Therefore, the con-similarity transformation to diagonalize the scattering matrix is

$$\left[S_{(\underline{X}_1, \underline{X}_2)} \right] = \left[U_{(\underline{X}_1, \underline{X}_2) \mapsto (\underline{X}, \underline{Y})} \right]^T \left[S_{(\underline{X}, \underline{Y})} \right] \left[U_{(\underline{X}_1, \underline{X}_2) \mapsto (\underline{X}, \underline{Y})} \right] \quad (62)$$

where, the unitary matrix $\left[U_{(\underline{X}_1, \underline{X}_2) \mapsto (\underline{X}, \underline{Y})} \right]$ takes the form

$$\left[U_{(\underline{X}_1, \underline{X}_2) \mapsto (\underline{X}, \underline{Y})} \right] = \begin{bmatrix} \underline{X}_1 & \underline{X}_2 \end{bmatrix} \quad (63)$$

and it can be parameterized as given by (37), i.e., it can be expressed in terms of the three angles $\{\phi, \tau, \alpha\}$. Huynen parameterized the resulting diagonal matrix as follows

$$\left[S_{(\underline{X}_1, \underline{X}_2)} \right] = m \begin{bmatrix} 1 & 0 \\ 0 & \tan^2 \gamma \end{bmatrix} e^{j\xi} \quad (64)$$

where m represents the maximum polarization and the angle γ is the target ship angle. Finally ξ consists of an absolute phase. As a result, Huynen parameterized the scattering matrix in terms of the 5 Euler parameters $\{m, \gamma, \phi, \tau, \alpha\}$ and the absolute phase ξ .

As is can be observed in **Figure 11**, the angle γ determines the angle between the line formed by the states $(\underline{X}_1, \underline{X}_2)$ and $(\underline{Q}_1, \underline{Q}_2)$, both considered to be over the same section or plane of the Poincare sphere. Additionally, the three Euler parameters $\{\phi, \tau, \alpha\}$ consist of the rotations to be done over every one of the three axis defining the space in order to bring the polarization state given by \underline{x} , corresponding to a linear polarized wave, to the polarization state given by \underline{X}_1 .

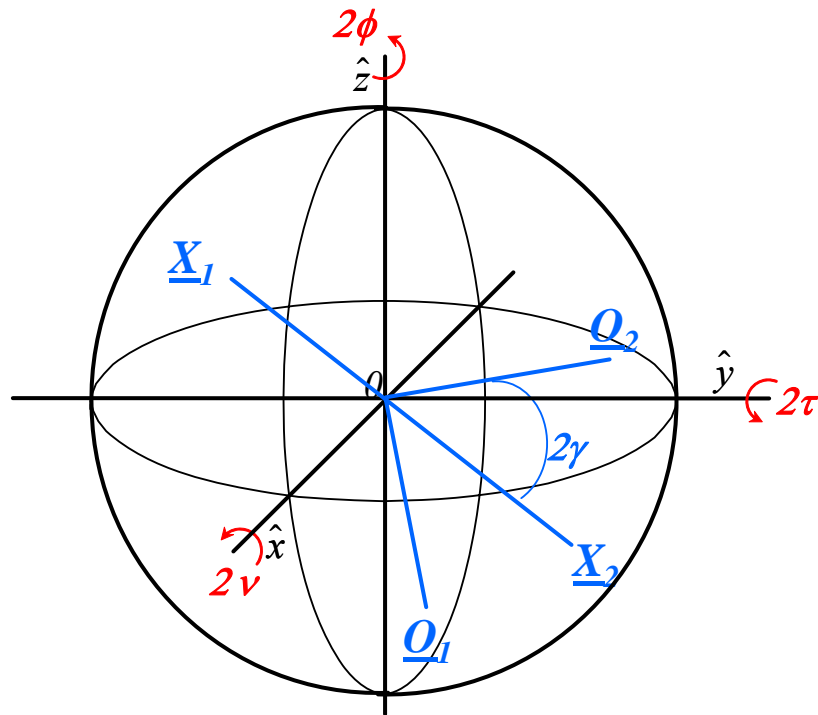


Figure 11 Polarization fork.

2.5 Canonical Scattering Mechanisms

A real target presents always a complex scattering response as a consequence of its complex geometrical structure and its reflectivity properties. Consequently, the interpretation of this response is obscure. As it shall be presented in a future chapter, a possible solution to interpretate this response is to decompose it into the response of canonical mechanisms. These scattering mechanism are characterized by presenting a simple scattering response. This section presents the list of this canonical scattering mechanisms and its characterization by the scattering matrix and the polarimetric signatures. The scattering matrix shall be presented in three different orthogonal polarimetric bases:

- Linear polarization basis (h,v) where h and v stand for the horizontal and vertical polarizations respectively.
- Linear rotated basis (a,b) where a and b stand for the 45 degrees linear and the -45 degrees linear polarizations respectively.
- Circular polarization basis (l,r) where l and r stand for the left circular and the right circular polarizations respectively.

2.5.1 Sphere, Plane, Trihedral

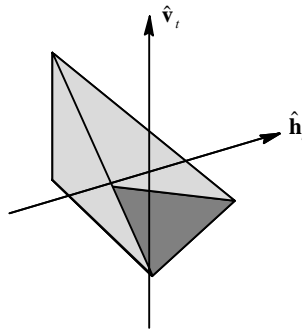


Figure 12 Trihedral orientated horizontally.

Scattering matrices of a sphere, a plane or a trihedral in the three polarization basis:

Linear polarization basis (h, v)

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Linear rotated polarization basis (a, b)

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Circular polarization basis (l, r)

$$[S] = \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix}$$

Co-polar and cross-polar signatures of a sphere, a plane or a trihedral:

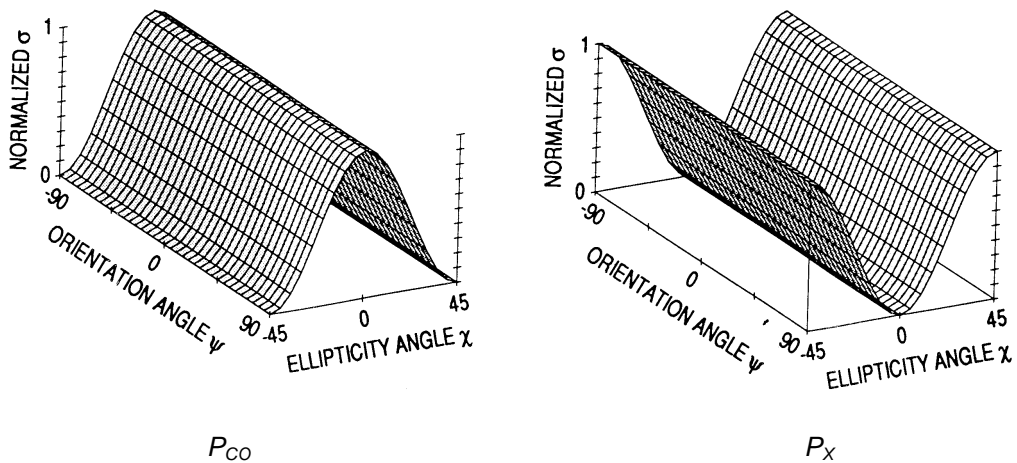


Figure 13 Co-polar and Cross-polar signatures corresponding to a sphere, plane or trihedral.

2.5.2 Dipole

2.5.2.1 Dipole orientated in the vertical axis

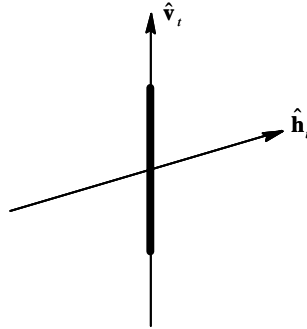


Figure 14 Dipole orientated in the vertical axis.

Scattering matrices of a dipole orientated in the vertical axis in the three polarization basis:

Linear polarization basis (h, v)

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Linear rotated polarization basis (a, b)

$$[S] = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Circular polarization basis (l, r)

$$[S] = \frac{1}{2} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

Co-polar and cross-polar signatures of a dipole orientated in the vertical axis:

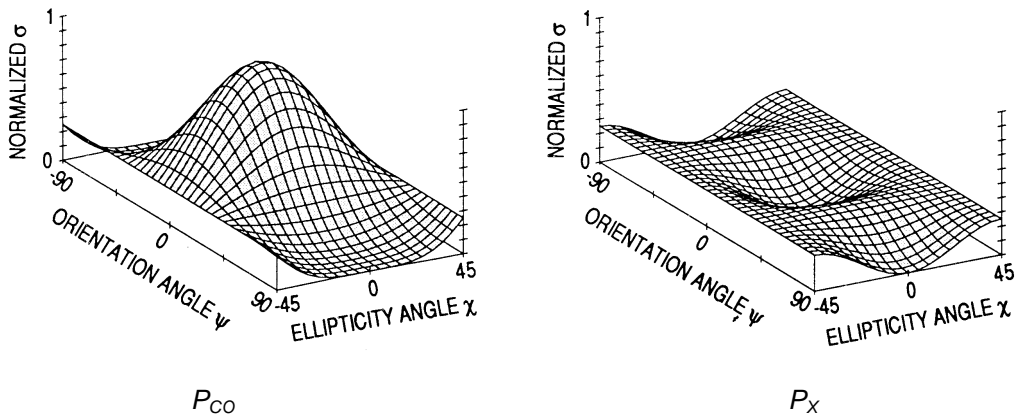


Figure 15 Co-polar and Cross-polar signatures corresponding to a dipole oriented vertically.

2.5.2.2 Oriented dipole

In this case, we consider the dipole oriented with an angle ϕ :

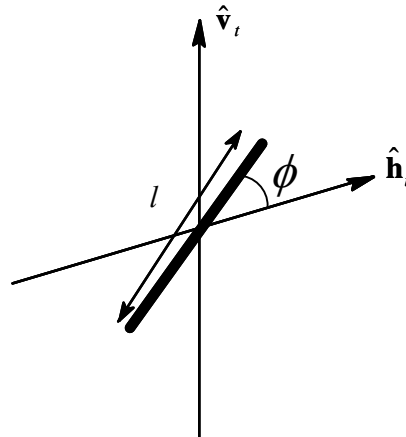


Figure 16 Dipole oriented with a ϕ angle.

Scattering matrices of a dipole oriented with an angle ϕ in the three polarization basis:

Linear polarization basis (h, v)

$$[S] = \begin{bmatrix} \cos^2 \phi & \frac{1}{2} \sin 2\phi \\ \frac{1}{2} \sin 2\phi & \sin^2 \phi \end{bmatrix}$$

Linear rotated polarization basis (a, b)

$$[S] = \begin{bmatrix} \frac{1}{2} + \cos \phi \sin \phi & \frac{1}{2} - \cos^2 \phi \\ \frac{1}{2} - \cos^2 \phi & \frac{1}{2} - \cos \phi \sin \phi \end{bmatrix}$$

Circular polarization basis (l, r)

$$[S] = \frac{1}{2} \begin{bmatrix} e^{j2\phi} & -j \\ -j & e^{-j2\phi} \end{bmatrix}$$

Co-polar and cross-polar signatures of a dipole oriented with an angle ϕ :

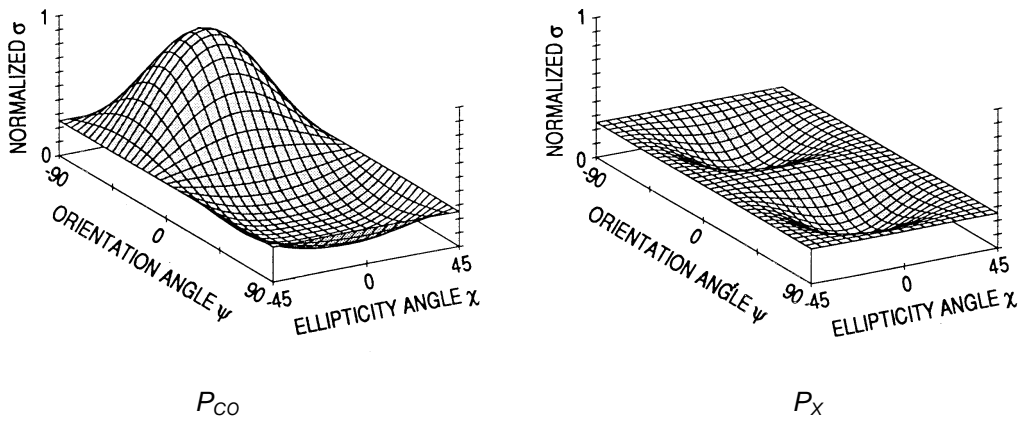


Figure 17 Co-polar and Cross-polar signatures corresponding to a dipole oriented with a ϕ angle.

2.5.3 Dihedral

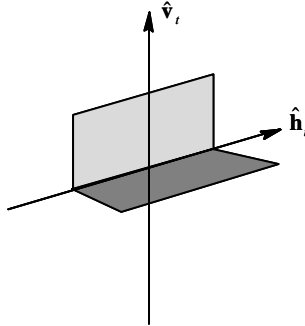


Figure 18 Dihedral oriented in the horizontal axis.

Scattering matrices of a dihedral oriented in the horizontal axis in the three polarization basis:

Linear polarization basis (h, v)

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Linear rotated polarization basis (a, b)

$$[S] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Circular polarization basis (l, r)

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Scattering matrices of a dihedral oriented with an angle ϕ in the three polarization basis:

Linear polarization basis (h, v)

$$[S] = \begin{bmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{bmatrix}$$

Linear rotated polarization basis (a, b)

$$[S] = \begin{bmatrix} \sin 2\phi & -\cos 2\phi \\ -\cos 2\phi & -\sin 2\phi \end{bmatrix}$$

Circular polarization basis (l, r)

$$[S] = \begin{bmatrix} e^{j2\phi} & 0 \\ 0 & e^{-j2\phi} \end{bmatrix}$$

Co-polar and cross-polar signatures of a dihedral oriented in the horizontal axis:

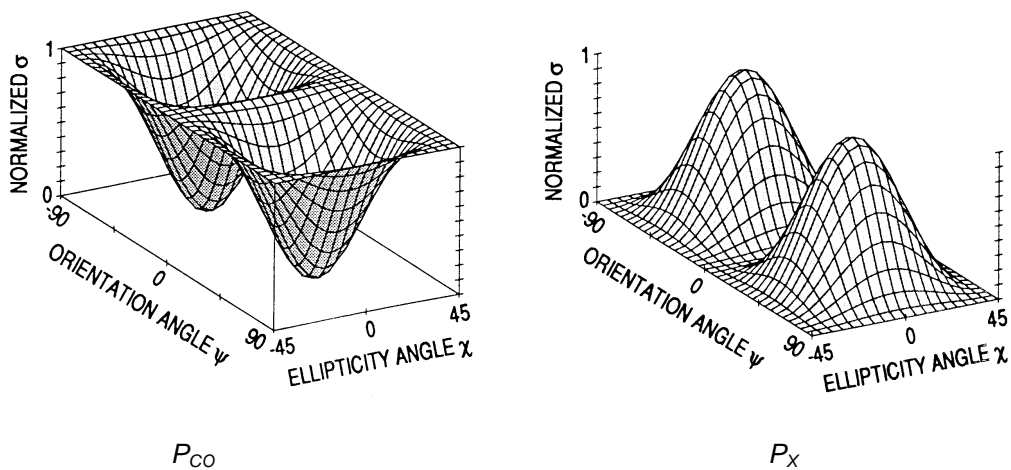


Figure 19 Co-polar and Cross-polar signatures corresponding to a dihedral oriented in the horizontal axis.

2.5.4 Right Helix

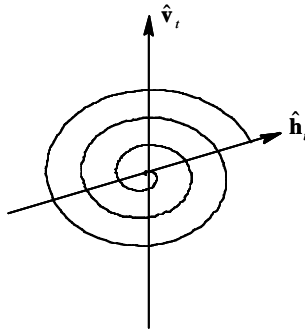


Figure 20 Right helix.

Scattering matrices of a right helix oriented with an angle ϕ in the three polarization basis:

Linear polarization basis (h, v)

$$[S] = \frac{1}{2} e^{-j2\phi} \begin{bmatrix} 1 & -j \\ -j & -1 \end{bmatrix}$$

Linear rotated polarization basis (a, b)

$$[S] = \frac{1}{2} e^{-j2\phi} \begin{bmatrix} -j & -1 \\ -1 & j \end{bmatrix}$$

Circular polarization basis (l, r)

$$[S] = \begin{bmatrix} 0 & 0 \\ 0 & -e^{-j2\phi} \end{bmatrix}$$

Co-polar and cross-polar signatures of a right helix oriented at 0 degrees:

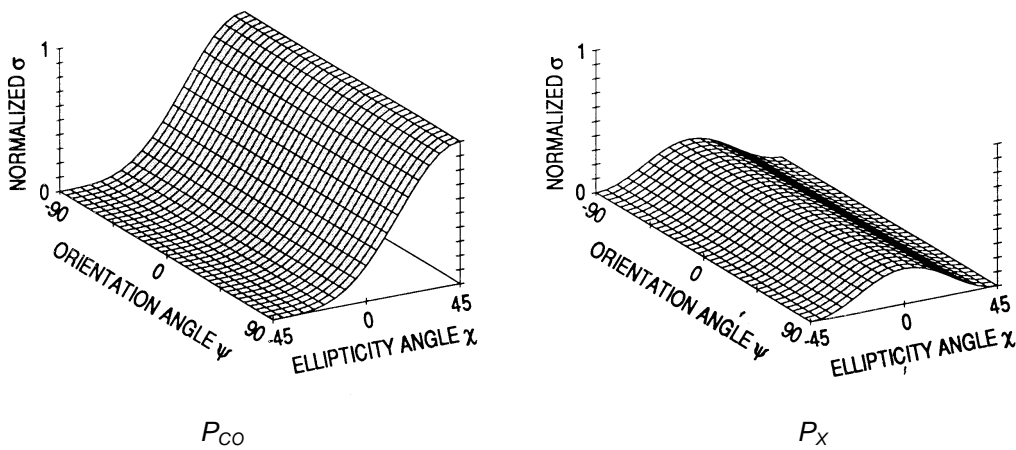


Figure 21 Co-polar and Cross-polar signatures corresponding to a right helix.

2.5.5 Left Helix

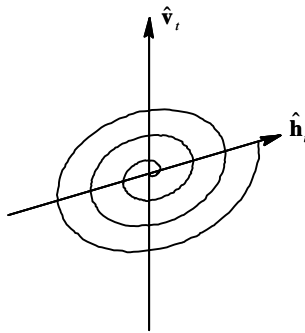


Figure 22 Co-polar and Cross-polar signatures corresponding to a left helix.

Scattering matrices of a left helix oriented with an angle ϕ in the three polarization basis:

Linear polarization basis (h, v)

$$[S] = \frac{1}{2} e^{-j2\phi} \begin{bmatrix} 1 & j \\ j & -1 \end{bmatrix}$$

Linear rotated polarization basis (a, b)

$$[S] = \frac{1}{2} e^{-j2\phi} \begin{bmatrix} j & -1 \\ -1 & -j \end{bmatrix}$$

Circular polarization basis (l, r)

$$[S] = \begin{bmatrix} e^{-j2\phi} & 0 \\ 0 & 0 \end{bmatrix}$$

Co-polar and cross-polar signatures of a left helix oriented at 0 degrees:

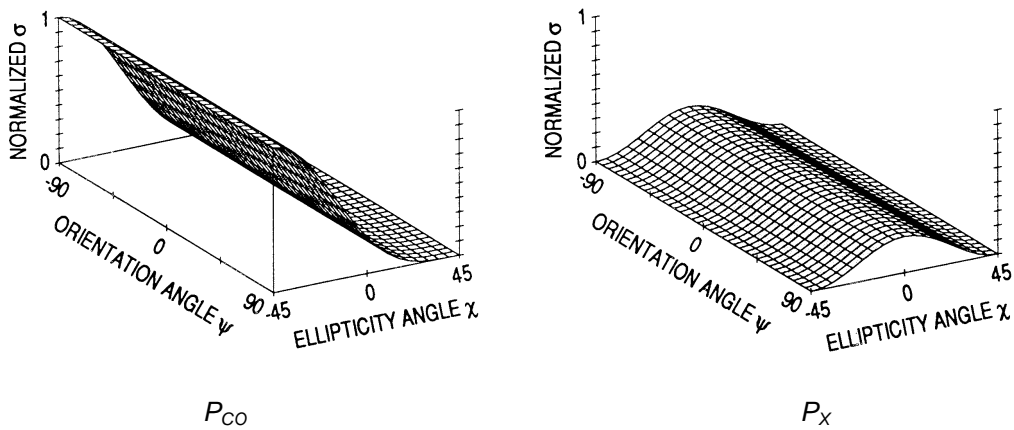


Figure 23 Co-polar and Cross-polar signatures corresponding to a left helix.